In contango versus backwardation, the truth may not be in convenience: disequilibrium states and the spot-forward balance in commodity markets

by

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Abstract

The notion of a stochastic convenience yield to explain variations and reversals in the spot-forward premium has become widely used in commodity market research and practice. However such variations may arise from causes more intrinsically related to the structure and cash flows of the extended commodity markets than to convenience as such. An instance is where the markets can be subject to disequilibrium phases, characterised by rationing or clearing impediments that interfere with arbitrage. These are likely to arise when market inventory is in short supply, so that disequilibrium switches can be based on the inventory:sales ratio. The disequilibrium methodology is applied to six base metals between 2005-2011, suggesting that lead, tin and nickel are susceptible. Risk and interest rates are also important determinants of the spot forward balance rate.

Key words: backwardation, base metals, commodity, contango, convenience yield, disequilibrium econometrics, regime dependence, risk.

JEL reference numbers: G12, D40, D53, C 24, C 58.

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1. Introduction

The relationship between spot and forward commodity prices has in recent years been thrown into the broader spotlight of public attention. In part this arose with high profile corporate crises; notably that of Metalgesellschaft AG, precipitated by a switch during 1992-3 of the oil market from a normal state backwardation, where forward prices are lower than spot prices, to contango\(^1\). A second major impetus has been the rise of China as an industrial power, together with a search by hedge funds for alternative investments after the global finance crunch and the enhanced spectrum of tradable products (e.g. carbon emission rights). Commodity trading has become big business. In turn, commodity spot and forward prices have become the subject of day to day media commentary. Switches between contango and backwardation have become more frequent (as in figure 2, section 4).

Academic and research attention has a long history, notably with the early writings of J.M. Keynes (1930). According to his theory, ‘normal backwardation’ arises because industrial hedgers have bias towards long rather than short forward positions. Speculators fill the gap by taking profitable short positions, with the risk offset by higher current spot prices. Habitat and risk therefore accounted for departures from situations where the cost of carry in the form of the interest rate would, otherwise suggest contango. A second strand of discussion arose with the idea of convenience yield, due initially to Kaldor (1939), broadened into a more general theory of storage by Brennan (1958) and subsequent authors. This extends the cost of carry to encompass storage and transactions costs, but also a ‘convenience yield’ to commodity users. The net cost of carry, factoring in the interest cost, can at times become negative, creating a situation where spot prices are higher than forward. The convenience yield concept has been refined by a number of recent authors. Dependence of the convenience yield on the current level of physical inventory has attracted special attention e.g. Raynauld & Tessier (1984), Fama &French (1987,1988), and Schwartz (1997).

Implications for basis variation or mean reversion have been explored by Bessembinder (1992), Casassus & Collin-Dufresne (2004), and for the extended commodity term structure Routledge et al (2000). The term ‘convenience yield’ itself has come to be regarded as a composite of all influences associated with the carry of inventory stocks; as distinct from the interest rate, which can be regarded as the cost of carry for money.

The present contribution adds a further dimension, that of disequilibrium dynamics. In a perfect world, the actions of agents with different motivations are reconciled with efficient pricing mechanisms by means of an institutional framework that may be implicit as well as explicit. A commodity market can be regarded as a compound structure with multiple clearing prices for each constituent market. Thus in addition to spot and forward prices for the commodity itself, there is an implicit market for the lending and borrowing of the commodity. The market for gold comes as close as anything to such an ideal framework. The gold loan rate adjusts to meet the demands of arbitrageurs who borrow gold to sell now to take advantage of a futures price seen as too cheap (section 2). The convenience that arises to users from having gold on hand, rather than lending it, becomes factored in the gold loan rate, as does the storage cost for the gold inventories necessary for such lending operations. In effect the convenience yield becomes a well defined objective reality, and the gold loan rate can exceed the interest rate as the cost of financial carry, leading to observed backwardation. Commercial gold storage with transferable (and lendable) titles has become big business with major repositories in Singapore, Switzerland and the US.

However, markets for most physical commodities as not so well endowed with structure, even where spot trading might be liquid. It may be very expensive or even unrealistic to establish large scale commercial storage facilities; among the base metals, lead is an instance. Alternatively, storage might exist but require search special knowledge or search costs to locate, especially where market inventory is running low. In such circumstances the market clearing price of commodity loans might either not exist, or be slow to equilibrate. At any point in time, one can observe spot and forward prices for the commodity, and an interest rate as the cost of finance. The rate that balances the three can be termed the spot-forward balance rate. In state of full equilibrium this becomes the equilibrium loan rate, equal in turn to the convenience yield. Out of equilibrium there is an adjustment process that might take time. Econometric models of market disequilibrium based on imputed clearing prices provide a useful empirical representation for such partial adjustment mechanisms (Bowden 1978a,b). In more less normal times full equilibrium holds, and spot and forward prices can be modelled in terms of expectations and risk. But a signal that disequilibrium is occurring can be based on the state of inventory, as the ratio of inventory to sales. Below some critical point for this ratio, the market switches into a disequilibrium state. At such times the market can move into strong backwardation. But what is being identified as a convenience yield is in reality just a notional spot-forward balance
rate that balances up the three prices actually observed, namely the spot and forward prices together with the interest rate.

The existence and extent of such regime dependent behaviour is examined in the context of base metal prices. The six chosen metals, namely aluminium, copper, lead, nickel, tin and zinc are of significant industrial importance and all have both spot and futures markets, though of varying liquidity. Of the six base metals, lead, nickel and tin shows the strongest evidence of disequilibrium behaviour, though copper figures in the period post the GFC. Disequilibrium aside, copper shows the most marked evidence of expectations and risk effects. Aluminium shows little systematic dependence.

The scheme of the paper is as follows. Section 2 establishes a stylised market structure within which equilibrium or disequilibrium regimes are to be embedded. Agents are indentified by their activities and motivations; thus there are producers and users operating via the spot market, speculators taking open bets on spot or forward prices, and arbitrageurs operating between them. These are functional descriptions; a given person or company could combine two or more of these roles. Within such a framework, equilibrium values and likely determinants can be established for spot, forward prices and the commodity loan rate. Discussion then turns in section 3 to disequilibrium states, where the apparent commodity loan rate, backed out from of the spot-forward balance as the ’convenience yield’, differs from the imputed equilibrium rate. Discussion moves on to connect these states with the forward premium actually observed. The estimating equation that results embodies expectations and risk effects, together with an additional term that switches in or out according to whether disequilibrium holds, identified in terms of the inventory:sales ratio. Section 4 contains the empirical work, which incorporates developing economic influences out of China. Section 5 concludes with a discussion and overview.

2. Agents, structures, and full market equilibrium
To the casual observer, commodity markets are simply places where people buy and sell commodities. However this is a misleading simplification, for a modern commodity market is peopled by agents of different motivations and economic interests, who may operate within several embedded sub-markets. This section sets up an efficient, if idealised, compound structure. Equilibrium values and their determinants can be utilised as a basis for later discussion of the impact of potential disequilibrium inefficiencies.

2.1 The agents
As with most such typologies, the distinctions involved are abstractions that are not exclusive, so that any particular player may at the same or different times adopt more than one position, or act with multiple motivations. With this qualification, our stylised agents are as follows:

(a) Producers or users. Markets exist in the first place because industrial users demand the commodity, with a corresponding supply response from producers.

(b) Physical storers. Storage will refer here to finished inventory, rather than in the production pipeline as goods in progress. It includes producer stockpiles, consumer stock in hand, and to stocks held by independent commercial storage operators and available for purchase or loan.

(c) Speculators. These are agents who take open positions in the forward market, gambles which may or may not have a physical spot leg. They are responsive to their expectations for future prices in relation to current spot prices, or to the term structure of forward prices.

(d) Arbitrageurs. The aim here is to take fully covered positions where spot and forward prices are seen as out of line.

Here and in what follows, let:

- \( S_0 \) = the current (time 0) spot price.
- \( F_0 \) = the current price for forward (period 1) delivery.
- \( r_g \) = the commodity loan rate.
- \( r \) = the current borrowing rate for money; it will be assumed in what follows that this is determined exogenously to the commodity market.

The agents characterised under (a)-(d) act within the three implied submarkets: spot, forward, and commodity loan markets. Gold comes as close as anything to such an idealised market, with a well established gold loan rate, in addition to spot and forward prices for gold itself.

Demand and supply in each submarket are linked by arbitrage activity. In the present paper arbitrageurs are seen as subject to potential constraints, which may encompass liquidity shortages, institutional or behavioural barriers. However, in the absence of any such constraints, the textbook arbitrage operation works via all three implied markets. If the forward price is seen as too cheap, then arbitrageurs will borrow the physical at rate \( r_g \) and sell it spot at \( S_0 \). The borrowed physical commitment is covered by buying on the forward market at \( F_0 \). Alternatively if the spot price is seen as too cheap, arbitrageurs will buy the physical spot, using money borrowed at rate \( r \); lend the physical at rate \( r_g \) and sell the proceeds forward. Appendix A provides a step by step summary of the transactions.
The net result is that the three implied prices will adjust to ensure that \( F_0 (1 + r_g) = S_0 (1 + r) \), which is spot forward parity. Rate \( r \) is the carry cost of the physical, while rate \( r_g \) is the effective carry cost of the forward.

The equilibrium commodity loan rate will reflect storage costs, including the opportunity cost of funding it. It will also encompass the convenience benefit to users from having stock on hand should it be needed to ramp up production. In a perfect market, the commodity loan rate can be equated with the convenience yield, effectively the net price of storage.

2.2 Commodity market structure

As earlier mentioned, a commodity market is typically a compound structure made up of three markets or submarkets: for the physical commodity, for forward delivery (or the equivalent non-deliverable), and for commodity loan agreements, which doubles as an implied storage market. All three markets are linked by flows. At any moment in time, the total expressed supply into the spot market is given by the sum of supply from opening stock, new producer supply; and also arbitrage supply if spot and forward prices are seen as out of line. Likewise, the total spot demand depends upon the flow of industrial user demand, any demand excess to current carryover, and arbitrage operations or arbitrage settlement demand outstanding from the previous period.

Figure 1 depicts a possible form of the connectivity, without being too prescriptive, as variants are possible. Flows are indicated with solid lines, causal influences with dotted lines. Important stock concepts are indicated with square boxes. Markets are indicated by circles. \( K_0 \) is opening stock held by all parties. It can either be supplied into the spot market (\( q^s_{k0} \)) or it can be allocated to the carry stock \( K_{c0} = q^c_{k0} \). Carry stock is available for arbitrage operations and it feeds forward into next period’s opening stock \( K_1 \).

The spot market occupies centre stage. Its role is to determine the current period 0 price \( S_0 \), and it is fed by three sources:

(i) \( q^s_{k0} \) = flow supply from opening inventory held by all agents, the latter denoted \( K_0 \) (far left box in figure 1). It is assumed in what follows that \( q^s_{k0} = \phi_k (S_0, S^c_1, r_g) K_0 \) with \( \phi_1 > 0, \phi_2 < 0 \); so that the complementary carry allocation \( q^c_{k0} = K_{0c} = (1 - \phi) K_0 \) is increasing in expected period 1 price \( S^c_1 \) in relation to the current price \( S_0 \).


(ii) \( q^*_0 = \text{new flow supply from producers: } q^*_0 = \varphi_s(S, z_s) \) where \( z_s \) refers to exogenous production drivers.

(iii) Supply from the associated commodity loan market originating with arbitrage operations. On the demand side, the spot market flows are:

(iv) \( q^d_0 = \text{demand by industrial users to support their own current production activities. It is assumed that } q^d_0 = \varphi_d(S_0, z_d), \text{ where } z_d \text{ incorporates exogenous demand drivers.} \)

(v) \( q^d_k = \text{demand expressed in the spot market by users and other parties for next period’s desired opening inventory } K_{1}^d. \text{ For the latter, we write } K_{1}^d = \kappa(S^f_1), \text{ so that high expected prices induce more investment in stock. Incremental demand will occur only if desired end of period inventory exceeds the carry } K_{c0}. \text{ In this case, } q^d_k = [K_{1}^d - K_{c0}]_+ = [\kappa(S^f_1) - K_{c0}]_+. \) The balance between the above supply and demand flows determines the spot price \( S_0. \)

The commodity loan market utilises carry stock \( K_{c0} \) as described in figure 1. The two flow directions (dotted lines) indicate that all spot market sales originating in arbitrage must be supplied out of carry stock and returned into carry at the end of the period. Expected future spot prices are an important formative input on the current forward rate \( F_0 \) which has a feedback relationship with the commodity loan rate \( r_g. \) In full equilibrium, the latter is determined within the arbitrage submarket. Arbitrage demand and supply, as expressed in the spot market, may be taken as proportional to any perceived spot-forward discrepancies.
2.3 Full equilibrium

A full equilibrium path will be defined as one along which spot forward parity continually applies, with no dynamic influences from current or outstanding arbitrage operations. Adopting more general time notation, let $S_t^*, F_t^*, r_{gt}^*$ denote such a path. The spot market balance reduces to

$$q_{k0}^c = K_{ct} + \phi_t(S_{t+1}^*, S_t^*, \pi_t) K_t^* + \phi_s(S_t^*, s_{sl}^*) = \phi_d(S_t^*, z_{dt}) + [\kappa(S_{t+1}^*, \pi_t) - K_{ct}]_+$$

where

$$K_{ct}^* = (1 - \phi_{k+}) K_t^*$$

and inventory accumulation is given by

$$K_{t+1}^* = K_{ct} + [\kappa(S_{t+1}^*, \pi_t) - K_{ct}]_+$$

Equilibrium forward prices are assumed to be driven by expected future prices and by one or more risk factors collectively denoted by $\pi_t$:

$$F_t^* = \phi(S_{t+1}^*, S_t^*, \pi_t) .$$

Along the equilibrium path spot-forward parity applies

$$F_t^* (1 + r_{gt}^*) = S_t^* (1 + r_t) .$$
Equations (1)-(3) determine the equilibrium time path for the spot, forward prices and the commodity loan rate.

A final remark concerns risk. In full equilibrium states, risk is identified with general volatility in future spot prices, as it affects producers and users. In practice, risk can also enter via a short squeeze where potential inventory sellers or lenders withhold stock to exploit known or suspected short forward positions. This is more likely to arise where market inventory is in short supply. States of this kind are the subject of the next section.

3. Disequilibrium states

The treatment that follows utilises the no-arbitrage equilibrium as a benchmark solution. Once this is established the modifying effect of spot-forward disequilibrium can be developed as modification to the benchmark solution. Disequilibrium itself is identified as a failure of spot-forward arbitrage associated with deficiencies or impediments in the associated commodity loan market. In some contexts, established loan markets may not exist. However even where they do exist, clearing may be incomplete or imperfect, problems often associated with the lack of a formal traded market or open outcry, in this case for loans as bilateral OTC agreements. In the borrower case (regime A of figure A1), an arbitraging borrower might be willing to pay more, only to find that his customary sources have already committed all available funds, at a lower rate. This might particularly arise where inventory available to lend is tight and search becomes difficult. Even in regime B an arbitrager who has purchased stock with the intention of lending it may well find that nobody wants to borrow. Such a situation might arise if there is already a lot of inventory in the market, so that even non-arbitrage demand for stock is minimal.

3.1 Non-equilibrating commodity loan rates

For the above reasons, search and clearing impediments for available inventory or inventory loans may create situations where from time to time, \( r_{gt} \neq r_{gt}^{*} \). In turn, this will induce out of equilibrium behaviour for spot and forward prices. Thus:

\[
\frac{S_t - S_t^*}{S_t^*} = \zeta (r_{gt} - r_{gt}^*) \\
\frac{F_t - F_t^*}{F_t^*} = \psi (r_{gt} - r_{gt}^*)
\]

with \( \zeta(0) = \psi(0) = 0; \; \zeta' > -1; \; \zeta' < 0; \; \psi' > 0 \). To see the effect of these conditions, suppose that there is loan rationing in regime A, so that \( r_{gt} < r_{gt}^* \). Arbitrageurs cannot
borrow all they want at the prevailing rate \( r_{gt} \). This implies a corresponding constraint on buying forward (so that \( F_t < F^*_t \)) and on selling spot (so that \( S_t > S^*_t \)). Hence the function \( \psi \) passes upwardly through the origin. A similar analysis applies to the spot disequilibrium; in this case the schedule \( \zeta \) has a negative slope through the origin.

For further reference three contingencies may occur, including equilibrium (regime E). They are as follows:

- **Regime A**: \( r_{gt} < r^*_t \) and \( \Delta_t = F_t (1 + r_{gt}) - S_t (1 + r_t) < 0 \)
- **Regime E**: \( r_{gt} = r^*_t \) and \( \Delta_t = F_t (1 + r_{gt}) - S_t (1 + r_t) = 0 \)
- **Regime B**: \( r_{gt} > r^*_t \) and \( \Delta_t = F_t (1 + r_{gt}) - S_t (1 + r_t) > 0 \).

For future reference it may also be noted that

- **Regime A**: \( \zeta > 0, \psi < 0; \; (\zeta - \psi) / (1 + \psi) > 0 \);
- **Regime B**: \( \zeta < 0, \psi > 0; \; (\zeta - \psi) / (1 + \psi) < 0 \).

### 3.2 The forward premium

The forward premium, usually written as \( F-S \), is a common object of observation and analysis in the commodity markets. In the implicit form

\[
(1 + r_{ct})F_t = (1 + r_t)S_t
\]

the ‘convenience yield’ \( r_{ct} \) has a proxy relationship with the forward premium, though with a negative sign:

\[
r_{ct} = r_t - \frac{(F_t - S_t)}{S_t} \approx r_t - \frac{F_t - S_t}{F_t}
\]

If market inventory is low, then its shadow price should be high, with a higher ‘convenience yield’. In such a circumstance the observed forward premium would be lower.

However, the term ‘convenience yield’ can be a misleading without further qualification. For as defined in expression (5), \( r_{ct} \) corresponds in form to a commodity loan rate. As such, it can inherit the separate influences of a market price, such as disequilibrium constraints in the processes setting the observed market price. For such reasons, we will refer to \( r_{ct} \) in more neutral terms as the ‘spot-forward balance rate’.

If \( r_{gt} \) is the currently observed commodity loan rate then from expressions (5) and (6)

\[
r_{ct} = r_{gt} - \frac{\Delta_t}{F_t}.
\]
In regime A, $\Delta_t < 0$. Thus the spot-forward balance rate exceeds the apparent commodity loan rate. To this extent, the spot forward balance rate can be regarded as a shadow price that more closely reflects unsatisfied supply of inventory for arbitrage purposes.

Finally, combining expressions (4) and (6) connects the spot-forward balance rate with the equilibrium loan rate in the form:

$$1 + r_{ct} = (1 + r_{gt}^*) \frac{1 + \zeta_t}{1 + \psi_t}.$$  

This can be further consolidated by making use of a summary decomposition of the equilibrium loan rate. Denote the time $t$ expected rate of change in equilibrium spot prices as

$$s^e_t = \frac{S_{t+1}^* - S_t^*}{S_t^*}.$$  

Consistent with expression (2), suppose also that

$$F_t^* = \frac{S_{t+1}^*}{1 + \pi_{ft}}.$$  

In expression (8b), $\pi_{ft}$ is a risk factor expressed in discount form (it can be of either sign). Higher values correspond to a higher risk premium. Using equations (8a,b) in conjunction with (3) gives

$$1 + r_{gt}^* = \frac{(1 + r_t)(1 + \pi_{ft})}{1 + s^e_t}.$$  

Combining (9) with (7) gives the spot forward balance as

$$1 + r_{ct} = \frac{(1 + r_t)(1 + \pi_{ft})}{1 + s^e_t} \left(1 + \frac{\zeta_t - \psi_t}{1 + \psi_t}\right);$$

or to a first approximation,

$$r_{ct} \approx r_t - s^e_t + \pi_{ft} + \frac{\zeta_t - \psi_t}{1 + \psi_t}.$$  

Expression (10b) can form the basis of an estimating equation, with the substitution of suitable proxies for the right hand terms. Operational aspects are discussed in the next section.
4. Empirics

The empirical application is to a set of base metals chosen for their industrial importance, for the liquidity of spot and forward markets, but different arrangements or ease of storage, and hence potential disequilibrium effects. Figures 2a,b depict the history of the forward premiums ($ (F - S) / F $) for aluminium, copper, zinc (fig. 2a) and for lead, nickel and tin (fig. 2b). It is apparent for all these metals that there are substantial variations over time, and switches between contango (positive values) and backwardation (negative values).

Figure 2a: Forward premiums for aluminium, copper and zinc
The London Metals Exchange (LME), which is the primary data source for the present study, has on its website\textsuperscript{2} a series of pie charts depicting the industrial uses of base metals. However, it could be argued that the primary drivers of price movements are in fact marginal users and new users, more so than overall (or intra-marginal) use patterns. Anecdotal evidence from the markets is that the economic rise of China has been pivotal in this respect, for Chinese demand now accounts for 35-40\% of world base metal usage. Copper and aluminium prices have been subject to both effects. Much of this use has been for electricity cabling associated with the building boom in China. Moreover, as copper prices have risen, the demand has grown for aluminium as a cabling substitute. While still a relatively minor segment on the pie chart of world uses, the use in China for cabling has become an important price driver for aluminium as well as copper. The importance of the Chinese economy is reflected in the expectational variables chosen for the present study.

4.1 Variable specification

Expression (10b), as an explanatory equation for the spot-forward balance, or the equivalent spot-forward premium, contains a number of conceptually distinct influences among the right

\textsuperscript{2} The industry uses pie charts metals follow e.g. http://www.lme.com/copper_industryusage.asp
hand variables. The immediate task is to represent these influences in realisable form for empirical testing. Detailed data sources are listed in section 4.2.

(a) The imputed expectations effect $s_t^e$ is captured by means of a two part proxy. The first is an effect common to all the metals. Two alternatives were explored, based on smoothed versions of either Chinese GDP or the Chinese purchasing manager (PMI) index for industry. Of the two, the Chinese GDP was retained as the preferred business cycle variable. We use the forecast of this trend as a common expectation variable in our framework. The smoothing algorithm used to establish the trend follows Hodrick & Prescott (1997) and Ravn & Uhlig (2002) with the Ravn-Uhlig smoothing parameter set at 129,600 for monthly frequency data. As a common effect, fitted coefficients may vary, even as to sign, according to extent to which the metal’s use pattern differs over the cycle. The overall effect is intended to capture business cycle responses where these may be common to all metals.

A second expectational proxy is specific to the metal concerned. Aeroplanes, cars and trucks are a major use of aluminium, and the US is the largest market with annualized car sales of an average of 14.5 million in 2011. For aluminium a US transportation index is therefore used. Although copper has many industrial uses, the largest demand during the last years stems from Chinese building, both residential and commercial. Hence a Chinese real estate index is employed for copper’s expectation.\(^3\) The main demand for lead is for batteries, and for this the US transportation index is used. Nickel is a major component of stainless steel and likewise, the demand for zinc correlates closely with the steel indices. The change in the Chinese steel index is used for nickel and zinc. Tin is used in electronic equipment and the semiconductor demand index from China is a natural proxy.

(b) A similar two part structure is used for the risk premium $\pi_f$. For the common effect based on general trader sentiment, we utilise two measures of global uncertainty. The first is the VIX index, which based on the implied volatility in CBOE options written on the S&P500 index. A second indicator is Libor the London interbank offer rate, which rises at times of general economic stress or uncertainty. The rationale for Libor is that its 2007-2008 collapse signalled riskier times ahead for the world economy. For the metal specific effect we utilise the volatility of the metal price computed on a moving window of 20 monthly observations.

(c) The last term on the right hand side of equation (10b) can be taken as a regime discriminator. From expression (5b), it will be positive in regime A (loan rationing) and in

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\(^3\) Copper is also used in the electro industry, however on a smaller scale.
regime B it will be negative. The maintained hypothesis will be that disequilibrium is associated with market inventory shortage, relative to sales. Thus only regime A is potentially applicable. In what follows, the ratio \( \frac{\text{inventory}}{\text{sales}} \) for the respective metals is taken as a switch to detect whether regime A is holding at any given time. For this purpose, a simple discriminator is constructed in terms of the function based on the 0-1 switch or ramp function. The constructed variable used is

\[
(11) \quad \frac{\text{inv}}{\text{sales}} \times RF(v - \frac{\text{inv}}{\text{sales}}),
\]

where

\[
RF(v - \frac{\text{inv}}{\text{sales}}) = 1 \iff \frac{\text{inv}}{\text{sales}} < v,
\]

\[
= 0, \text{ otherwise.}
\]

According to expression (11), the regime A rationing variable is inactive once the inventory to sales ratio rises above the critical point \( v \) to be estimated. Below that point, further tightness of the inventory ratio has a progressively rising effect on the spot forward balance (or loosely, convenience yield). The limitation, if binding, gets worse as market inventory declines, relative to sales. This is taken as the disequilibrium variable; the sales and inventory figures are those specific to each metal. The switch point \( v \) is a parameter to be estimated. A significant \( v \) could be taken as evident of regime dependent

(d) Finally, the left hand variable in equation (10b) is the spot forward balance, taken as

\[
r_{ct} = r - \frac{F_t - S_t}{F_t}.
\]

However that the same interest rate \( r \) also appears on the right hand side of the same equation, with unit coefficient. In order to improve conditioning, the equations are therefore fitted in term of the forward premium \( \frac{F_t - S_t}{F_t} \) alone; this will change the sign of the fitted right hand side coefficients, but is otherwise equivalent. Note from (b) above, that an interest rate, in the form of Libor, will nevertheless appear on the right hand side of the equation to be estimated, but as a common effect risk variable.

4.2 Data

Spot and forward time series for each traded metal are sourced from the London Metals Exchange in the form of monthly daily official close prices. The data span is 1999-2011 . The LME lists three different aluminium products traded, from which we focus on the plain aluminium contract only. We obtain the full term structure of quoted prices and estimate the
forward-premium as difference between the cash and the first forward month contract, i.e. the three month rolling forward. In practice, the forward-premium is in fact often calculated using the nearest future contract as proxy for the spot. Chinese business cycle data are sourced from Bloomberg Data Service.

Expression (11) for the regime A disequilibrium introduces a nonlinear element to the equations. For this we used maximum likelihood with Huber (1967)-White (1980) cluster-robust standard errors. A preliminary OLS fit with the inventory to sales ratio entering linearly (i.e. without the switch) is utilised to generate initial values. All variables are specified to be I(0), supported with Dickey-Fuller tests.

4.3 Results
The fitted versions of equation (10b) are displayed in tables 1 and 2. Table 1 is the fit for the full data period 2005-2011, while table 2 spans the later sub-period 2008-2011. The intent of the latter is to examine if changes might have occurred during or after the global financial crunch. This was a period that also saw considerable activity, if not volatility, in metals trading, stemming from a flight to alternative investment for hedge funds and other traders, on the back of Chinese economic activity, unaffected at that point in time.

Expected coefficient signs as listed in the table; note that these adjust according to the way the dependent variable is fitted (in this case as the forward premium \( (F - S) / F \)). An ambiguous sign is denoted [+,-]: for example, a given metal spot price might either lead or lag a common business cycle indicator. The model is nonlinear but the last row of the table gives quasi-\( R^2 \) values, which are generally good relative to prior experience with forward price ratios as dependent variable.

An overview of the results over the full sample period (table 1) suggests that lead, nickel, and tin form a subgroup of the base metals, with more or less the same predispositions as to the forward premium influences. For each metal there is a specific volatility effect, either significant or near significant at 5%, but little sign either of a single common volatility effect or else expectational influences, save possibly for tin.

Disequilibrium behaviour is signalled for lead and nickel, which are sensitive to both the regime indicator and to increasing tightness of the inventory ratio below that point. The regime indicator \((v)\) is certainly significant for tin, but its value (at nine times sales) seems too high. Disequilibrium effects may be associated with storage impediments with lead as the obvious instance, or market liquidity, which is not strong at times. Historically, the copper market has enjoyed superior liquidity, both spot and forward. Consequently it is not
surprising to find no evidence of disequilibrium effects, over the complete period. On the other hand, expectational and risk effects are more marked in the case of copper with strong significance for both Libor and specific volatility effects. Zinc and aluminium are problematic, either from statistical insignificance or counterintuitive signs.

Some differences appear when the sample period is limited to the post GFC period 2008-2011 (table3). Copper now shows some evidence of disequilibrium behaviour, while lead and nickel lose theirs. Aluminium now shows a specific volatility induced risk effect; a possible reason arises from the increased liquidity of the aluminium market. Libor now has the predicted positive significance. Post the GFC, a rising copper price induced Chinese builders to start substituting aluminium for copper in cabling. A more substantial volume of supply took place out of the day to day spot market rather than the more traditional longer term contractual arrangement for this metal.
Table 1: Fitted coefficients and asymptotic t values, full period

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<th></th>
<th>Aluminium (1)</th>
<th>Copper (2)</th>
<th>Lead (3)</th>
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**R^2** 0.411 0.805 0.496 0.655 0.471 0.582

**Time Span** Jan 2005 – Dec 2011
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</tr>
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Table 2: Fitted coefficients and asymptotic t values, post GFC
5. Concluding remarks

The notion of a stochastic ‘convenience yield’ has become a much used device in commodity modelling. However it is subject to criticism on the grounds of incomplete specification as to origins or causes. It might well arise for reasons that have little to do with the convenience of anybody in particular. A similar point is made by Jarrow (2010), who identifies embedded scarcity and usage options in the embedded cash flows. The present paper is to this extent similar, that it seeks an explanation in the way the markets work and the cash flows involved. The convenience yield can best be regarded as a balancing item that originates in the structure and operations of the market. As such there is a case for renaming it as the ‘spot forward balance rate’ or a similar label more neutral as to implied causation.

The spot forward balance rate may encompass a number of conceptually distinct items. In particular it may be expected to arise when inventory is tight in relation to sales. It may be worse when markets are more illiquid or storage is difficult, as with lead. And to the extent that it can be regarded as the disequilibrium outcome of a market imperfection, the variation in the spot forward balance, as we call it, is episodic in nature. That may mean it is less amenable to representations in terms of standard stochastic models. Our results confirm these conjectures. Episodic disequilibrium phases and their representations provide a rationale for the diminishing marginal effect as inventories rise, noted by Fama and French (1987,1988) and others.

The results of the present paper confirm previous studies that find price volatility is an important determinant of the forward premium and the spot forward balance. Interest rates (for money) are an essential element of the spot forward balance, though in principle should cancel out when the equations are fitted in terms of the forward premium. However we do find that Libor continues to play a role in the forward premium as such. We attribute this to a risk signalling role associated with the collapse of Libor at the end of 2007. Other dependences are more specific to the metal concerned. There is some evidence for the industrial rise of China as having an impact on the forward premium, in the case of copper, aluminium and lead. In this respect the results support what appears to be conventional wisdom in the commodity markets, where the demand for copper and aluminium is concerned.
Appendix A

Figure A1 summarises the arbitrage operations in a perfectly functioning market. The intent is to capture the precise cash flows entailed, and to illustrate that three sub-markets are involved in the determination of forward and spot prices, and hence the forward premium.

Arbitrage operations

Provisionally assume no storage or convenience costs/yields. Interest rate is $r$, commodity loan rate is $r_g$. Assume simple mid rate pricing (e.g. borrowing rate = lending rate). No credit risks.

(A): Forward price seen as too cheap: $F(1 + r_g) < S_0(1 + r)$

- Borrow 1 unit of physical at rate $r_g$, commit to remit $(1 + r_g)$ oz at end of period (EOP).
- Cover EOP reimbursement commitment by buying forward $(1 + r_g)$ units at $F$.
- Sell the 1 oz of borrowed physical on spot market; invest proceeds $S_0$ at rate $r$.
- EOP profit will be $S_0(1 + r) - F(1 + r_g)$

(B): Forward price seen as too expensive: $F(1 + r_g) > S_0(1 + r)$

- Buy 1 unit of physical on spot market at price $S_0$;
- Fund the purchase by borrowing $S_0$ at interest rate $r$.
- Lend the 1 unit of purchased physical at gold loan rate, will receive $(1 + r_g)$ units at EOP
- Sell the $(1 + r_g)$ units forward at price $F$, will receive $F(1 + r_g)$ at EOP.
- EOP profit will be $F(1 + r_g) - S_0(1 + r)$

References


