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Sustainability benchmarks for the sovereign debt to GDP ratio

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Abstract

'Too much' sovereign debt helped to precipitate the current economic crises in Europe and elsewhere. But how much is too much? Distinctions between sustainability and super-sustainability are explored with the aid of a benchmark formula that incorporates key parameters of public debate. The benchmark can used to inform remedial action, or as a basis for social bargaining as between social spending and growth imperatives. The need for associated equity reserves is a further constraint on sovereign debt sustainability. This can arise from the property that a sustainable benchmark is in generally dynamically unstable under economic shocks. It might also reflect desirable leverage in funding social infrastructure projects.

Key words: Public debt, financial crisis, sustainability tradeoffs, government spending, political bargaining, leverage, eurozone crisis.

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1. Introduction

The sustainability of sovereign debt has occupied centre stage in the economic crises of the eurozone countries. Too much sovereign debt might variously be viewed either as an incidental exposure of a proposed rescue package, or the cause of the crisis in the first place. But what is 'too much'? Legislators and economic commentators most often refer to the ratio of government debt to GDP as a single summary index of sustainability. A rule of thumb seems to be that a government debt to GDP ratio of 30% is workable, 60% is worrying, 90% is dangerous, and 130% is a crisis. For example, the Maastricht Treaty convergence criteria for aspiring eurozone members, as reinforced by the Stability and Growth Pact 1997, specified 60% for the Debt to GDP ratio, together with a side constraint of 3% of GDP for the annual government deficit. On the face of it, a single figure such as the debt to GDP ratio seems misplaced; for the capacity of a government to service its debt should depend upon other things, the prevailing rate of interest being just one of them. But there is merit in simplicity as a focus for public debate and decision making. Most economists would agree that government debt can be a good thing, if it is used to finance productive infrastructure as a public good, or alternatively as a buffer against temporary economic shocks. The difficulty lies in a more precise call as to how much, and in turn, how to present this to policy makers in terms of just a few key decision parameters.

The present note suggests a simple benchmark formula that can serve as a common basis for public discussion and political bargaining between the forces of the left (who might favour social spending) and those of the right (who might prefer less tax or else more growth oriented policies). The framework can be used in alternative ways, depending on context. Thus one could feed in current values for core government spending, tax rate, growth rate, and interest rate, to examine whether the current debt to GDP ratio is sustainable, and what would be required to fix the problem. Or in a longer term context, the sustainable debt ratio could be used as a quasi constitutional limit to govern political bargaining around arguments such as core social depending and tax rates. The same framework can be used to broaden the debate to issues of debt leverage in financing public infrastructure. The issue here is whether the public sector should be subject to the same principles that govern capital structure for private sector companies. As the eurozone crisis is foreshadowing, bankruptcy costs can evidently arise for sovereigns as well.

Literature approaches on public debt span a number of distinct lines of enquiry. Earlier authors debated issues of the general welfare in the context of general growth theory, within

contexts such as private savings displacement, or disincentive effects of the taxation needed for servicing. Thus Meade (1958) took a negative view, save for the public infrastructure effect; similarly Diamond (1965). More recently, empirical work has considered the correlation between government debt (as normalised by GDP) and economic growth rates. A common finding suggests that the relationship is nonlinear: good up to a point and bad beyond (Reinhart and Rogoff (2010), Kumar and Woo (2010), Checherita and Rother (2010)). The empirical relationship with interest rates has been explored by e.g. Paesani *et al* (2006). A possible adverse externality in this respect is the 'crowding out' effect on interest rates and capital formation (Elmendor and Mankiw (1996), Agenor and Montiel (1996)). Krugman (1988) coined the phrase 'debt overhang' to refer to the depressant effect of non sustainability on economic activity. The determinacy issue has further been considered by a number of recent authors within the context of optimising macroeconomic models, e.g. Aiyagari et al (2002), Schmitt-Grohe & Uribe (2004), Adams & Billi (2008), Diaz et al (2008), Adams (2010), Reinhart & Rogoff (2010)). Treatments differ in assumptions about household production and rigidities. In most such models, the government relies on explicit taxation to service its debt, rather than implicitly via inflation, which together with interest rates may fall under the aegis of an independent central bank. The government issues nominal debt and is the dominant player, with taxpayers as followers in the implied game. Intertemporal maximising households with constant time preferences consume and allocate labour subject to technology shocks. Tax rates influence labour supply and resulting solution is not first best. A general result is that government debt in a Ramsay optimising model follows a random walk around a rising trend, in this respect reinforcing Barro (1979). However the trend itself is indeterminate, expressed in different levels of government debt relative to GDP, and preferences regarding tax or interest rates, and their social consequences. One way to derive an (apparently) more precise metric is to run a computable general equilibrium (CGE) model through with an assumed set of projection input numbers. This has its own demands; it requires a hypothesised economic structure, parameter calibrations, and projected future states of the exogenous drivers. Moreover, such models and their assumptions are 'black box' so far as the general public are concerned. An additional source of potential distrust is the spin that may be put on the numbers by the body that commissions them, often the very same government that is the object of the exercise. In other words, both complexity and transparency are issues.

In addition, it could be pointed out that the process for setting targets as to public spending and debt is a political one, in which the interests of a diverse set of stakeholders and

interested parties have to be reconciled. Models that assume a homogenous set of consumers and producers, and a unitary public utility function for the government, are not well adapted to agent heterogeneity, with tradeoffs and bargaining as part of the agreement process. What appears to be lacking, as a basis for informed public debate and consensus seeking, is a focus on simplicity. In principle, whether or not a given amount of public debt is sustainable would seem to depend on just few key decision and state variables: GDP as an index of capacity to pay, the rate of interest, the tolerance for government spending on debt servicing, and the economy's rate of growth, all come to mind.

The most immediate application is prudential in nature. The formula might suggest that along a balanced growth path, and given long run averages for interest rates or debt servicing ratios, the sustainable debt to GDP ratio is 50%; as against the current ratio of 100%. But perhaps the current ratio is perfectly affordable in the short term, so there is no special need to buy sovereign credit default swaps, sell the local currency, or dismiss the government. But if the public, guided by the economic media, know that in the long run only the 50% is sustainable, they may come to accept that remedial action needs to be implemented. In this respect, the proposed benchmark formula is concerned with sustainability, rather than optimality. It represents an equilibrium relationship into which commentators can read their own preferences or politicians their bargaining point. There is scope to allow differences as to which of the parameters they assign priority. In this sense the proposed thumbnail formula can be readily reversed in its policy impact reference. One commentator might prefer to think of the debt to GDP ratio as the primary object, and whether it is excessive in its relation to current levels. Another might consider whether, and how much, state equity financing should be used to complement the public debt financing of social infrastructure. The proposed formula offers a starting point of reference for such discussions. Optimality, in such terms would be more appropriately viewed within framework of Nash bargaining between political interest groups.

Debate of this kind presupposes that there is in fact a problem. High growth economies can exhibit a property called 'super-sustainability', where any notional path of public debt can be financed. This gets a bad press from potential analogies with Ponzi schemes, for it represents a violation of the classic transversality condition. It is nevertheless a happy state for the economy to be in, for a notionally sustainable debt path is in general dynamically unstable. The latter property is a further motivation for public equity reserves in more mature economies.

The scheme of the paper is as follows. The basic debt to GDP sustainability ratio is derived in section 2, motivated in terms of the standard transversality condition. The distinction between sustainability and super-sustainability is made in this context. The extent to which selected countries satisfy this condition is briefly illustrated. It is noted that sustainability is itself is not a buffer against economic shocks; indeed the same condition leads to stochastic instability. Potential use of the public debt benchmark as a bargaining tool is developed in section 3, encompassing adverse parameter feedback effects. The issue of public debt leverage ratios is treated in section 4. Reserving equity to back debt is a desirable response to economic shock contingencies, even where steady state sustainability might exist. The same risk imperative also arises in financing public infrastructure, where the need to retain public equity can further diminish the sustainable debt to GDP ratio. Section 5 contains some concluding remarks.

2. The long run balance equation

The discussion that follows starts with an exposition of the spending-debt balance equation and sustainability conditions. A basic benchmark formula emerges in terms of just four state preference parameters or state variables, namely the government core funding proportion, the aggregate tax rate, the rate of economic growth, and the interest rate. A distinction is drawn between sustainability and super-sustainability. Some brief case studies are presented using development to that point. Instability under shocks is noted, followed by the role of the benchmark in disequilibria associated with corrective action.

2.1 The basic balance

In what follows, government expenditure (G) and debt level (D) are measured in nominal dollars, euros, etc. Government debt is assumed state independent (i.e. not state contingent), and carries an interest rate[2] (r) that is taken provisionally as exogenous. A key decision parameter is the government debt service ratio, derivable by adjusting the primary budget balance for public debt servicing costs. Thus if θ is the proportion of G committed to current spending on goods, services & transfers, the complement $1-\theta$ is the proportion devoted to servicing the public debt. On the income side, aggregate tax income is measured as a

[2] To allow for the possibility that the government might wish to raise debt via zero coupon bonds, or with a deep discount issue of coupon bonds, the rate of interest is taken the yield rather than a nominal coupon. One could imagine the government issuing a multi-year zero coupon bond this year (year *0*), and repaying the capital sum the following next year (year 1) out of a new zero coupon issue with the same maturity. On rollover the government would then be paying the yield on the time *0* issue.

proportion τ of GDP. In the presumed steady state, GDP is specified to grow at rate *g*, the 'balanced growth rate'. Which parameters are to be taken as given data, and which are to be the subject of policy bargaining or prescription, is at the discretion of the user. Parameters τ,θ, and g can have alternative interpretations, depending on context. Thus they could be based on historical averages 'over the cycle', or as targets reflecting a preferred state of the economy and public spending. Alternatively, θ and τ could be regarded as bargaining parameters between political interest groups. Interpretations and possible interdependencies are explored in subsequent sections.

Key variables and balance equations are as follows:

T = total tax take = τGDP.

G = Sovereign government spending (includes central government investment).

D = current stock of government debt.

NG = *G* - *rD* = government spending net of debt servicing.

$NG = \theta G$.

$\dot{D} = G - T$ = Government debt accumulation.

The foregoing imply that

(1) $$\frac{\dot{D}}{D} = \frac{r}{1-\theta} - \tau\frac{GDP}{D}.$$

In a steady state growth path, it is assumed that GDP grows at rate *g*:

(2) $$\frac{G\dot{D}P}{GDP} = g\ .$$

Specify that along this steady state balanced growth path, the government debt to GDP ratio is to remain constant. From expressions (1), (2) the sustainable debt to GDP ratio is given by

(3) $$\delta = (\frac{D}{GDP})^* = \frac{\tau(1-\theta)}{r - g(1-\theta)}\ .$$

Expression (3) defines the sustainable growth path for government debt. Written in flow terms as

(4) $$(r - g(1-\theta))D^* = \tau(1-\theta)GDP^*\ ,$$

the left hand side says that the effective interest burden on servicing the debt is diminished by the economy's growth rate, while the right hand side refers to the funds available to service the debt.

A determinate value for the debt to GDP ratio (δ) exists for $g(1-\theta) < r$. Such a condition corresponds to the usual definition of public debt sustainability, which refers to the finite present value of debt. Solving forward the differential equation (1) gives:

$$D_0 = e^{-rT/(1-\theta)} D(T) + \tau \int_0^T e^{-rs/(1-\theta)} GDP(s) ds .$$

The second right hand term refers to the present value of the tax revenue used to service the debt up to time *T*. The usual sustainability requirement ('transversality condition') is that the first right hand term tends to zero as $T \to \infty$:

(5) $\lim_{T \to \infty} e^{-rT/(1-\theta)} D(T) = 0 .$

If this were not the case then the burden of current debt could be shifted forward *ad infinitum*; public debt would have become a Ponzi scheme, or a 'debt bubble', amounting to a potential free lunch for the current generation at the expense of future generations. Now along the proposed balanced growth path, *D(t)* is supposed to grow at the same rate *g* as *GDP(t),* so as T becomes large, $D(T) \propto e^{gT}$. Hence the transversality condition will hold along this path, provided $r > g(1-\theta)$ as specified in connection with expression (3).

On the other hand, if $r < g(1-\theta)$ then the interest bill of any level of borrowing can be met out of economic growth; one could call this 'super-sustainability'. Along the economy's growth path, the public debt to GDP ratio can be any desired number. Alternatively, any targeted ratio would be more than met, in the sense that the debt balance would tend to zero. The government might instead elect to raise the proportion θ of revenue going to social services. There might be nothing short sighted or improper about such a choice. Indeed, the semantics of sustainability have to some extent become confused with the negative connotations of the 'Ponzi' label. Under super-sustainability it can be quite in order for the current generation to enjoy superior welfare benefits, knowing that the funding burden on the next generation will be at worst transitory. Thus super-sustainability deriving from superior economic growth may be regarded as a happy state for an economy to be in.

Section 3 explores stability issues arising from the distinction. In the meantime, it will be assumed that the focus of concern is for economies that are at risk of violating the sustainability condition in the wrong direction; so that $r > g(1-\theta)$, but the current debt level may be too large in relation to GDP.

The sustainable debt ratio varies directly with the effective tax rate on GDP, and inversely upon the interest rate *r*. In addition,

$$\frac{\partial \log \delta}{\partial \theta} = -\frac{r\delta}{(1-\theta)^2} < 0 .$$

A higher proportion of government spending devoted to goods and service means a lower proportion to servicing debt and a lower sustainable debt to GDP ratio.

Remarks:

1. The sustainable debt ratio (expression (3)) refers to a balance along an equilibrium growth path of GDP and government spending. As such, it can be presented with an alternative target, so that parameters can be reversed. For example, if the steady state sustainability ratio δ is pre-specified, then the implied proportion of tax revenue going to debt servicing is given by

(6) $$1-\theta = \frac{\delta r}{\tau + \delta g} .$$

This can also be useful in pointing out the allocation consequences of the current debt to equity ratio; an example is given below.

2. The question might arise as to whether the analysis should be carried out in nominal or real terms. Budget balance equations leading to expression (1) have to be formulated in nominal terms. However for given real rates, the rate of inflation will have a bearing on the nominal rate of interest and the growth rate. In turn, this means that the sustainable debt ratio will be different in an economy committed to following a zero inflation regime. Thus suppose that the rate of inflation is constant at π along the steady state path. So if r_* and g_* indicate the real rates, then $r = r^* + \pi$; $g = g^* + \pi$. Assume that real and nominal θ, τ ratios remain the same. Let δ_0 denote the sustainable debt to equity ratio in a world where inflation was zero (the 'zero inflation ratio'), and let δ denote the ratio if the inflation rate is $\pi > 0$. Applying formula (3) shows that $\delta_0 > \delta$; the lower inflation regime entails a higher sustainable debt to GDP ratio. In terms of the servicing balance (4),

$$(r_* - g_*(1-\theta) + \pi\theta)D^* = \tau(1-\theta)GDP^* .$$

For a given real coupon r_* and growth rate g_* , higher inflation means a higher effective nominal coupon rate on the left hand side.

2.2 Worked examples

Some brief case studies, taken from current experience (end 2011), will serve to illustrate how the proposed sustainability ratio can be used.

Example 1: Italy

For Italy, the proposed tax rate τ = 0.435GDP is based on a 2008 OECD report. The current value of θ can be estimated on the basis of figures quoted in the European Commission's

2011 Spring Report: a primary budget surplus of 0.75% of GDP, but a deficit of 4% once interest payments are incorporated. So NG = (0.435-0.0075) GDP; T-G= - 0.04 GDP.

Hence $\theta = \frac{NG}{G} = \frac{(0.435 - 0.0075)GDP}{0.43GDP + 0.04GDP} = 0.909574 \sim 0.91.$

Assume g is 5% in nominal terms and a current interest rate of 7% (medium term note at Dec.2011). Inserting these numbers into expression (3) would result in $(\frac{D}{GDP})^* = 60.1\%$. In such terms, the current Italian debt to GDP ratio of 120% is clearly not sustainable. Or using expression (6), the implied core expenditure ratio reduces to only 83% of total government expenditure.

It could be argued that the parameters as given above are those from stressful times, and not those of a steady state. Using instead the historical average of 1.7% for the primary budget surplus, and a long term interest rate of 4.5%, would raise θ to 0.94, more into line with other countries. But this combination results in a sustainable debt to GDP ratio of 59.8%, which is roughly the same, at about half the current level.

Example 2: New Zealand

For NZ, ten year averages suggest $\theta = 0.97$, $\tau = 0.48$, $g = 0.05$, and $r = 5.5\%$, the latter based on a five year maturity bond. This combination gives $\delta = 26.9\%$ as a sustainable direct public debt to GDP ratio. At the current (Nov. 2011) five year yield of 3.8%, the sustainable debt to GDP ratio rises to 39.5%, which is much the same as the existing actual ratio. However, some caveats would then be in order. NZ is a chronically a high interest rate economy, in both real and nominal terms, so the current rate of 3.8% is probably unrepresentatively low. Second, this is a small open economy subject to shocks which have recently become quite literal in nature, with earthquakes still ongoing. The debt to GDP ratio is projected to worsen as the bills come in for the rebuilding of Christchurch following the recent earthquakes of 2010-11. If this, or an outbreak of populism, required debt financed public spending such that $\delta =$ 100%, sustainability even at $r = 4\%$ would require $\theta = 92.45$, i.e. 7.55% of the tax take would have to go towards servicing central government debt. If this were to be judged unacceptable, either existing public services would have to be sacrificed to make way for the new programmes, or new sources of tax revenue found.

Example 3: The US

A review of ten year historical averages for GDP growth, together with US central government spending and debt, suggests parameters $g = 0.05$, $\theta = 0.9$, $\tau = 0.2$. At an

historical interest rate of 4%, this would result in $\delta = 57.1\%$. For an interest rate of 2%, the sustainable debt to GDP ratio would rise to 133.3%. Current US sovereign interest rates (averaging less than 1%) are even lower. The 2010 US central government debt to GDP ratio was 61%. This suggests that current government rescue bailouts are not excessive in terms of the movements required to restore sovereign debt sustainability.

For future reference, the 'sustainability margin' will refer to a buffer between the actual and sustainability debt to GDP ratio; conversely for a 'sustainability gap'. For the numbers quoted above, the US has a sustainability margin of (61% - 57%) = 3%. Italy has a margin of (60% - 120%) = -60%, so a sustainability gap of 60%.

2.3 Sustainability versus stability

The obvious rationale for preserving a sustainability margin is that it buffers the economy against adverse shocks. In this context, sustainability does not automatically imply stability, in the technical sense. To see this, compare the actual accumulation of debt to that under the sustainability benchmark, assuming the latter exists. The two equations are respectively

(7a) $$\dot{D} = \frac{r}{1-\theta} D - \tau GDP$$

(7b) $$\dot{D}_* = \frac{r}{1-\theta} D_* - \tau GDP_* .$$

Now suppose that there are shocks to GDP, measured relative to the steady state growth path, represented by $\varepsilon_y = \frac{GDP - GDP_*}{GDP_*}$. Also let $\Delta = \frac{D - D_*}{D_*}$ be the extent to which the actual debt level deviates from that under sustainability. Combining (7a,b) gives

(8) $$\dot{\Delta} = (\frac{r}{1-\theta} - g)\Delta - \frac{\tau}{\delta}\varepsilon_y .$$

It is now apparent that the sustainability condition does not guarantee stability under shocks. Indeed, to the extent that $r > g(1-\theta)$ the differential equation (8) is technically unstable, so that given any negative shock to GDP, debt diverges more and more from its sustainable benchmark. On the other hand, the dynamics (8) will always be stable for super-sustainability, where $r < g(1-\theta)$. Intuitively, a high growth economy can always finance its way out of interest servicing problems arising from a temporary income shock. The difference between sustainability and super-sustainability therefore refers to the inability of the former to self-correct for a shock without special institutional rules or features. As the mature economies typically have lower growth rates, special attention must be made to

developing or preserving such natural buffers as may exist. The role of public equity, to complement public debt, is one such feature. Section

2.4 Disequilibrium and the role of market perceptions

The steady state sustainable equilibrium framework can be used to describe what might happen in disequilibrium departures, as in the recent eurozone crisis. The key is a perceived gap between the sustainable and the actual sovereign debt to GDP ratio.

Figure 1 illustrates the ensuing dynamics of correction. Suppose an opening steady state is characterised by an interest rate $r_0 = 5\%$. A populist government is elected which is pressured to increase the proportion of core government spending from θ_0 to θ_1. With unchanged interest rate at $r_0 = 5\%$, the new sovereign debt to GDP ratio is marked as point B. But at θ_1, the sustainable debt to GDP ratio is now point C. A gap BC has opened up. Market perception is that the government is spending and borrowing too much, so interest rates rise to $r_1 = 7.5\%$, say. The sustainable debt ratio for this is at C_1, so the gap between the current and what is seen as sustainable is now even larger at BC_1 .

At this point, market confidence is imploding and interest rates exploding, to the point where either the government takes remedial action, or bailouts become necessary. Government budgets have to be trimmed back, and the core spending parameter θ is reduced. In turn, this raises the debt to GDP ratio perceived as sustainable. The disequilibrium gap between actual and sustainable starts to diminish. As market fears ease, the interest rate drops. If the corrective action succeeds, and the spending ratio θ is restored to the original θ_0, the interest rate has likewise fallen back to the original r_0, validating θ_0 as sustainable.

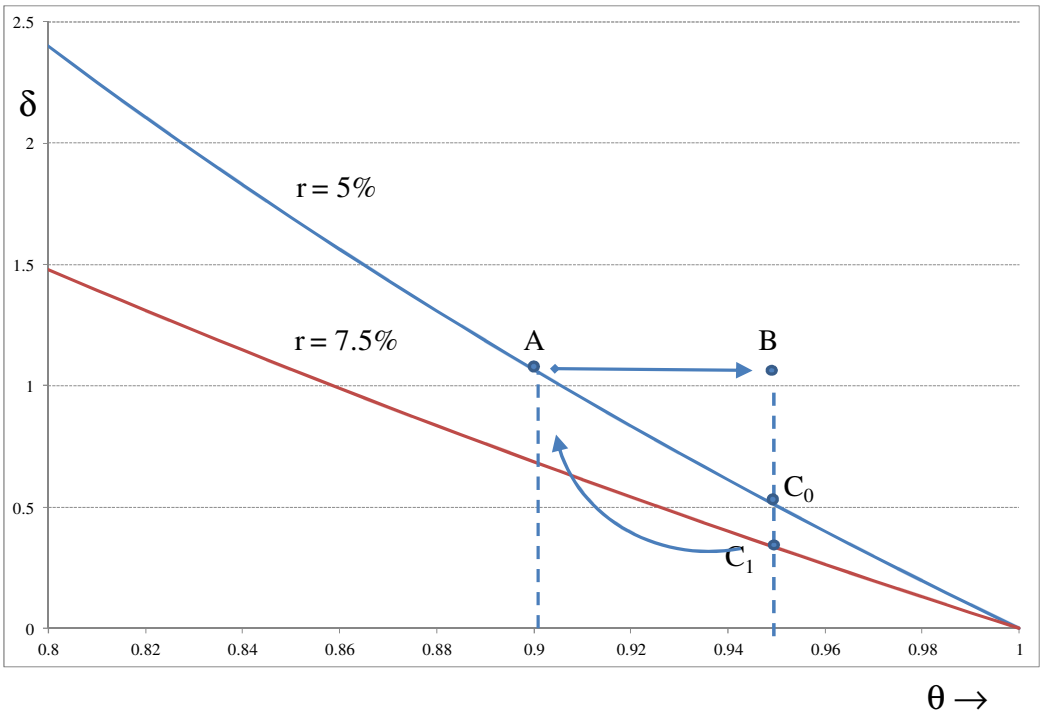


Figure 1: Crisis dynamics

3. Social bargaining and parameter tradeoffs

Steady state balanced growth paths can usefully be thought of as benchmarks for social bargaining. Politicians of the left are typically preoccupied with socioeconomic distribution; they will favour budgets that enhance welfare oriented spending and higher θ, in the preceding framework. Those of the right are more growth oriented; they will favour policies that reduce taxation τ and enhance the growth rate, *g*. Incompatibility arises because any measure that increases θ , the proportion of government income going to social spending (rather than interest servicing), will require an increase in the tax rate τ in order to preserve debt sustainability. The sustainability parameter δ can be thought of as a constitutional constraint, in much the same way as the 60% Maastricht limit quoted in section 1.

Figure 2a illustrates, with the provisional assumption that the growth *g* and interest rate *r* parameters remain unchanged at 7.5% and 5% respectively. The horizontal axis represents the disposable income proportion $1-\tau$, while the vertical axis measures θ. The solid line curves represent tradeoffs between θ and τ at two alternative sustainability limits, δ=50%, δ=75%. Any combination (θ,τ) lying above the chosen limit is unsustainable. Thus point P is sustainable for δ=50%, but not for δ=75%. The latter is a higher debt to GDP ratio that could only be serviced by either increasing taxes τ or reducing the proportion θ of social spending.

If now there is an agreed limit of 75% on debt, but it is proposed that the social spending ratio is to be raised from 92% to 94%, this will require taxes to be raised from 41.25% to 56.9% of GDP (moving from A to B along the 75% debt sustainability tradeoff).

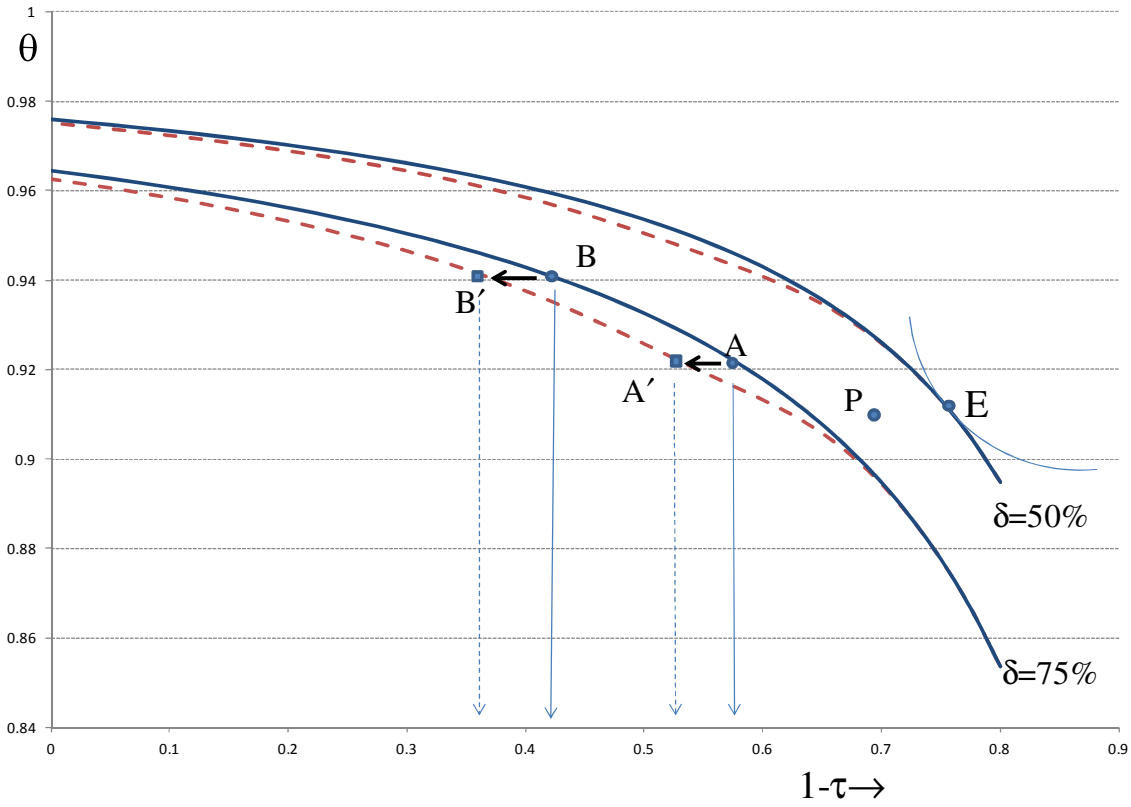


Figure 2a: Sustainability tradeoffs with economic growth feedbacks

However the party of the right could claim that the tradeoff is worse than this. Higher tax rates will act to depress economic growth, even without any interest rate effect. Thus suppose $g = g(\tau); g'(\tau) < 0$. Then along any given tradeoff curve,

$$\left.\frac{d\tau}{d\theta}\right]_{\delta = const} = \frac{1}{(1-\theta)^2} \cdot \frac{r\delta}{1+\delta g'(\tau)}.$$

If $g'(\tau) < 0$, then the tax rate must increase by more to sustain an increase in social spending.

Figures 2a,b taken jointly, illustrate the effect. Tax rates less than 20% are presumed to have no effect on economic growth, while at the other end a tax rate of above 60% of GDP is presumed to depress economic growth to zero (or so the Right will argue). Between these two limits the relationship[3] is negative sigmoid as in figure 2b. The two dotted curves in figure 2a

[3] The curve is obtained using a Binomial distribution function with n=32, p= 0.75.

are the resulting tradeoffs. The effect is to raise the tax rate required to support any given social spending ratio (e.g. A′ as compared to A). The effect arises because a required rise in tax will depress economic growth, requiring an additional tax hike to compensate. Likewise, if the social spending proportion θ is to be raised from 92% to 94%, this will require a 20 point increase in the tax rate, which is greater than the 15 point rise where the growth rate is invariant to the tax rate[4].

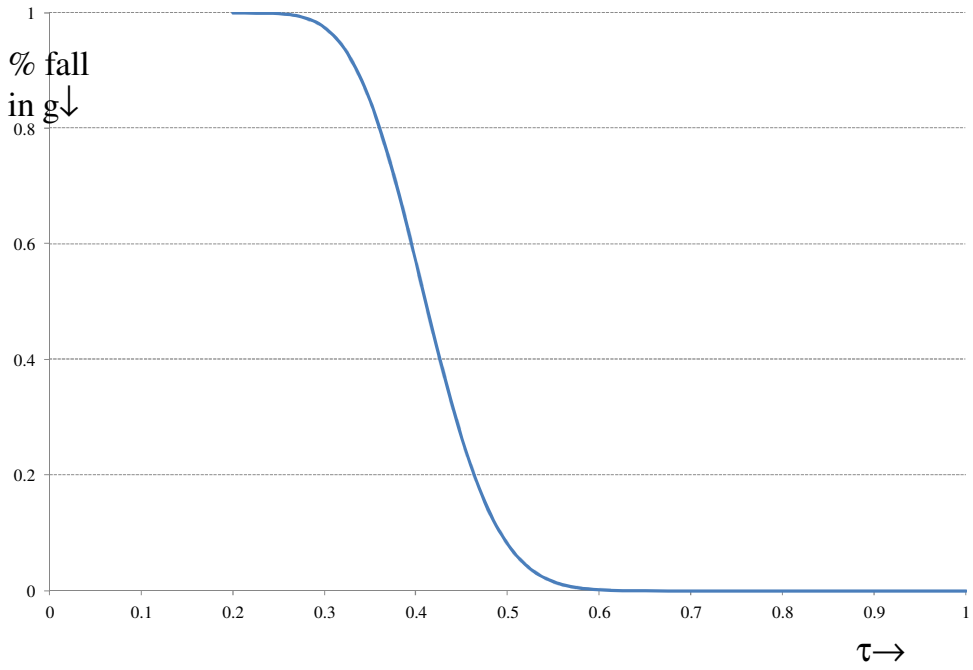


Figure 2b: Effect of tax rates on growth

The form of figure 2a corresponds to a notional Nash equilibrium cooperative game between the political forces of the left, who favour social spending (higher θ), and those of the right (lower taxes τ). Given a constitutional debt sustainability constraint δ=50%, the Nash bargaining solution might a point such as E. the precise location of this point will depend upon relative bargaining power and the non cooperative base solution[5].

[4] The discussion abstracts from compositional effects. For instance, a capital gains tax, where none existed before, might serve to discourage risk taking. This is especially true where the tax is levied on unrealised gains but no credit is claimable on subsequent losses. Thus even if compensated by a lowering of the personal tax rate, so that overall tax rate τ remains the same, the effect may be to limit growth and hence diminish the sustainable debt to GDP ratio.

[5] The Nash cooperative solution maximises the utility product $(u_L - u_{Lb})^{\lambda}(u_R - u_{Rb})^{1-\lambda}$ where u_{Lb}, u_{Rb} denote the fallback non-cooperative solution; $0 < \lambda < 1$ is a power weighting parameter.

Finally, the party of the right might maintain that a rise in the social spending ratio could induce higher interest rates, on the grounds that it would leave less of a safety cushion for adverse shocks to the government tax base. This serves to introduce a further general consideration, namely the desirability of reserving part of the tax take as government equity, with the twin objective of buffering shocks and to provide investor comfort for government bonds. Section 4 takes up the latter aspect.

4. Debt versus equity: infrastructure and risk

As indicated in section 2.3, more mature economies inherit an exposure to shocks that is not automatically compensated by the natural buffer of a high growth rate. To this extent, the government is subject to just the same financial discipline as the most private sector companies. Thus the present section is concerned with exposures to risk of various kinds, beginning with the risk associated with social infrastructure financing (§4.1) and extending to the risk of natural or financial shocks (§4.2). In both cases there is a role for public equity in absorbing such shocks.

4.1 Infrastructure financing

In the presumed steady state, government debt would be issued only to fund public sector investment, on the grounds that social infrastructure (IFS) is a public good that promotes economic growth and hence the future revenue base of the government. With a longer term project payoff period, the use of debt shifts the burden of servicing on to those who will benefit. However, it could be argued that public sector projects should be funded by equity as well as debt. One reason is that the economic payoff to such projects might well have inherent uncertainty. This could arise where the benefits are state contingent, or where the economic reasoning or forecasting underpinning the projects could be flawed in one way or another[6]. If so, then the present generation should arguably bear the risk. Moreover, an equity component would provide comfort to the buyers of debt and therefore lower the cost of capital for any given project. The upshot is that it is necessary to consider the government as sequestering part of its cash flow for the purpose of providing IFS equity.

A possible formalisation is to regard the public debt as issued by a government infrastructure entity (GIFS), perhaps nominal in character, with the government committing a steady flow of matching equity in the form of preference shares (Bowden and Lorimer

[6] A celebrated example incorporating both aspects was the Think Big projects of the early 'eighties in New Zealand e.g. http://en.wikipedia.org/wiki/Think_Big The core of the proposal was to use NZ iron sands as the basis for a steel industry, with further mineral spin offs such as titanium. Think Big became known to posterity as Sink Big.

(2008). At any point in time, the GIFS would have a given leverage ratio, conveniently denoted by the equity:debt ratio e = E/D. The equity component is valued at historic cost to incorporate annual budget allocations from the government. For simplicity it is supposed that the same cost of capital (*r*) applies to both debt and equity components. The GIFS fund earns the asset cost of capital on its investments paying out *rD* to its bondholders and *rE* as equity return back to the government for the equity capital contributed. The GIFS investments generate a public good component of private sector income, which we can formalise with an implied notional earnings flow T_{IFS} to the fund. Together with residual income tax T_0, the two elements make up the actual tax flow $T = T_{IFS} + T_0$. Figure 3 depicts cash inflows and outflows for the GIFS and the central government sponsor.

Cash flow equations for the central government and IFS fund are respectively:

(7a) $T_{IFS} + \Delta D + \Delta E = T_{IFS} + rD + rE$

(7b) $T + rE = \Delta E + NG$.

Consolidating gives the cash flow to government activities as

(7c) $\Delta D = NG + rD - T$.

In addition, total government spending is defined as

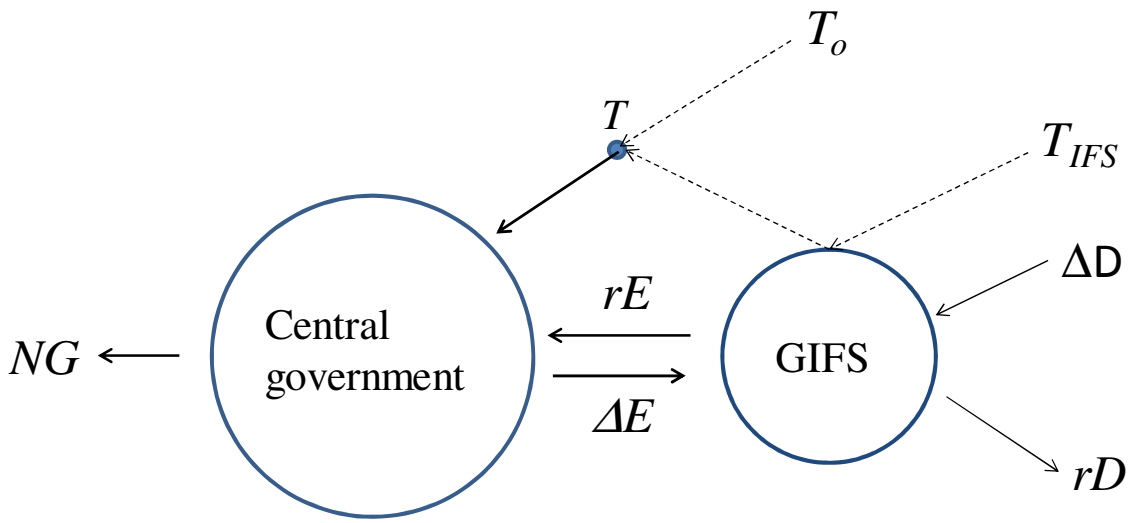
(7d) $G = \Delta E + NG + rD$.

As before, it is assumed that $NG = \theta G$ and $T = \tau GDP$.

In the steady state, D and E grow at the same rate *g*, and it is assumed that the IFS equity:debt ratio $(E/D) = e$, constant. The resulting sustainable debt to GDP ratio is given by

(8) $$\delta = \frac{\tau(1-\theta)}{r - g(1-\theta) + e\theta g}.$$

Comparing expression (8) with expression (3), it is evident that a requirement to preserve equity in public sector infrastructure will diminish the sustainable debt to equity ratio. For if the core spending ratio θ is fixed, then funds devoted each year to new equity will diminish the money available to service the interest flow on debt. The extent to which this is true depends on the size of θ. In the earlier quoted case of NZ, if the preferred IFS equity to debt ratio *e* = 33%, and θ remains at 97%, the sustainable debt to GDP ratio falls from 38% to 31%. But it may be that a policy of backing public debt with equity enables funds to be raised at a lower rate of interest. In this case the sustainable debt to GDP might actually be higher.



4.2 Equity as a buffer to economic shocks

From time to time, the government of the day will have to cope with contingencies that may be reasonably foreseeable in nature, if not in timing. Earthquakes are an instance. Countries bordering tectonic plate boundaries, such as New Zealand (the 'Shaky Isles'), Japan and Chile, might prudently set aside reserves to cover rescue and rebuild operations. Indeed, NZ already does so, through the NZ Earthquake Commission and the Natural Disaster Reserve Fund, which covers the initial excess on reinsurance payouts. But following the 2010-11 Christchurch earthquakes, the resources of the EQC were far exceeded, likewise those of some private insurers, with consequent drains on the public purse as last recourse. Nor do natural disaster rescues limit the contingent exposures of government. Some are financial in nature, as with the global financial crisis, where governments responded with financial institution bailouts, which may be executed directly rather than through their central banks. Severe economic recessions could be viewed within a similar contingent exposure framework. Equally, sovereign debt holders will be well aware of such contingent exposures and commitments. So the issue becomes how much equity to set aside to absorb potential shocks, and to provide equity backing for debt to be issues in exceptional times, as well as the more normal times envisaged in the preceding section.

Contingency provision amounts to the government being required to set aside equity reserves, or else the equivalent in capacity to raise further debt. One approach to working out how just much is needed might be to estimate an annual insurance premium to cover private insurance, treat this as an implied cost of equity capital, and capitalise it up to find the

required equity E_q. This would then be added to the equity needed to underpin IFS development. Expression (8) then applies, with the equity to debt ratio incorporating both equity elements: thus $\hat{e} = (E_q + E_{IFS}) / D$.

Reconsidering the NZ example (§2.2), suppose the GIFS equity ratio was 33%, while the contingency E_q ratio was calculated to require half of this, a further 16.5%. The result would lower the sustainable debt ratio from 31% to 29%.

5. Concluding remarks

Government debt is like many culinary ingredients; too much spoils the flavour. The problem that has been so evident, whether in media commentary or official communiqués, lies in deciding just how much is too much. In this respect, complexity has its costs. The purpose of the present paper is to propose, as a basis for public discussion, a relatively simple formula cast in terms of a few basis parameters. Some of the latter can be regarded as decision parameters, representing tradeoffs in political bargaining. Welfare consideration will always require the bulk of public spending to be devoted to current purposes of which a growing amount, year by year, is devoted to welfare related spending. The core funding ratio (θ) includes such elements, and this in turn will require taxation (τ). On the other hand if debt capacity is cut back, either from insufficient funds set aside to service it, or because the taxation required depresses economic growth, then the sustainable debt capacity has to fall; the more so because interest rates will rise, much as does the cost of capital for a low growth company. It is this sort of tradeoff that lies behind the coalitional and other bargaining games as between the left and the right of the political spectrum.

The analysis of section 4 has further highlighted another analogy with private sector corporate finance, namely the need for equity to back debt. The government leverage ratio, in other words, is just as important as it is in determining the private sector cost of capital.
It is also worth remarking that the sustainability parameters, and judgements based thereon, have to be set within the institutional environment of the economy as a whole. In this respect, negative synergies between public and private debt have received some attention from the credit rating agencies. A possible channel to public debt sustainability, as in the present model, is via the interest rate factor. Sovereign debt interest rates will be higher if the economy as a whole has a heavy dependence on offshore financing and hence an appreciable foreign exchange risk premium. New Zealand is a case in point: until recently the bulk of home mortgage lending originated indirectly via Uridashi and eurodollar funding, rather than

domestic savings. On a secular basis, NZ sovereign interest rates have as a consequence been higher than in other developed countries.

Such differences highlight whether or not it is desirable to be too prescriptive about sovereign debt sustainability numbers. There plainly should be limits, but whether these should be uniform across countries is dabatable, even in mature economies. For one thing, countries differ in the functions devolved to central government as distinct from state or local governments; this will affect inter-country core funding ratio comparisons. Limits might further depend upon whether the country concerned is part of a wider currency bloc, so that it cannot unilaterally solve a debt crisis by default and devaluation. Block solidarity would then require both a uniform and more restrictive sustainability limit, after which communal corrective action would become necessary. The social stress that this creates calls for wide public understanding and agreement up front as to what their public debt sustainability means, together with an understanding of the economic and political tradeoffs necessary secure it.

References

Adams, K., and R. Billi (2008) Monetary conservatism and fiscal policy, *Journal of Monetary Economics*, 55, 1376-1388

Adams, K., (2010) Government debt and optimal monetary and fiscal policy, Mimeo, Mannheim University.

Agenor, P-R., and D.L. Bevan (2005) *Development Macroeconomics*, Princeton: University Press.

Aiyagari, S.R., A. Marcet, T.J. Sargent and J. Seppala (2002), Optimal taxation without state contingent debt, *Journal of Political Economy*, 110(6), 1220-1254.

Barro, R.J. (1979 On the determination of public debt, *Journal of Political Economy*, 87(5), 940-971.

Bowden, R.J., and D.E. Lorimer 'The NZ term structure: Going long in infrastructure', *JASSA: The FINSIA Journal of Applied Finance* issue 2008 (3), 24-29.

Checherita, C., and P. Rother (2010) The impact of high and growing debt on economic growth: An empirical investigation for the Euro area, European Central Bank Working Paper No. 1237.

Diamond. P. (1965) National debt in a neoclassical growth model, *American Economic Review*, 55(5), 1126-1150.

Diaz-Gimenez, J., G. Giovannetti, R. Marimon, and P. Teles (2008) Nominal debt as a burden on monetary policy, *Review of Economic Dynamics*, 11, 493-514.

Elmendorf, D. and N. Mankiw (19990 Government Debt, in Taylor, J. and M. Woodford, *Handbook of Macroeconomics*, vol 1C, 1615-1669, Amsterdam: North Holland.

Krugman, P. (1988) Financing versus forgiving a debt overhang: some analytical issues, NBER Working paper 2486.

Kumar, M. And J. Woo (2010) Public debt and growth, IMF Working Paper 10/174

Meade, J.E. (1958) Is national debt a burden?, *Oxford Economic Papers*, New Series, 10(2), 163-183.

Paesani, P., R. Strauch and M. Kremer (2006) Public debt and long term interest rates: the case of Germany, Italy and the US, Working paper 656, European Central Bank, available on http://ssrn.com/abstract_id=913327

Reinhart, C.M. and K.S. Rogoff (2010) Growth in a time of debt, NBER Working paper No. 15639

Schmitt-Grohe, S. And M. Uribe (2004) Optimal monetary and fiscal policy under sticky prices, *Journal of Economic Theory*, 114(2), 198-230